
Warm-Up

Divide using Long Division

$$(3p^3 - 4p^2 + 5p - 6) \div (p - 1)$$

Divide using Synthetic Division

$$(x^4 + 4x^3 - 4x^2 - 22x + 20) \div (x - 2)$$

- Unit 4 Test

Objectives

Today we will...

- Use the Remainder Theorem to evaluate a function
- Apply the Factor Theorem to determine if something is a factor of a polynomial
- Apply the Factor theorem and our factoring techniques to find Zeros of polynomials

Remainder Theorem

If a polynomial is divided by $(x-k)$, then the remainder is $f(k)=r$

$$p-1=0 \\ p \neq 1$$

$$3p^3 - 4p^2 + 5p - 6 \div p-1$$

$$R = -2$$

Ex. 1 $(v^3 + v^2 - 9v - 24) \div (v + 2)$

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -9 & -24 \\ & & -2 & 2 & 14 \\ \hline & 1 & -1 & -7 & \textcircled{-10} \\ & & & & R \end{array}$$

$$(-2)^3 + (-2)^2 - 9(-2) - 24$$

$$-8 + 4 + 18 - 24$$

$$\textcircled{-10}$$

$$\boxed{f(-2) = -10}$$

Ex.2 Evaluate $f(x) = 3x^3 + 8x^2 + 5x - 7$ for $f(-2)$.

$$\begin{array}{r} -2 \overline{) 3 \ 8 \ 5 \ -7} \\ \underline{ 6 \ 16 \ 10 \ -14} \\ 0 \end{array}$$

-9

$$3(-2)^3 + 8(-2)^2 + 5(-2) - 7$$

-9

$$f(-2) = -9$$

Factor Theorem

A polynomial has a factor $(x-k)$ if and only if $f(k)=0$

(or using the remainder theorem... the remainder is zero)

Is $(x+2)$ a factor of $f(x)=2x^3+7x^2+7x+2$?

$$\begin{array}{r|rrrr} -2 & 2 & 7 & 7 & 2 \\ & & -4 & -6 & -2 \\ \hline & 2 & 3 & 1 & 0 \end{array}$$

yes, $(x+2)$
is a factor

$$\begin{aligned} & 2(-2)^3 + 7(-2)^2 + 7(-2) + 2 \\ & -16 + 28 - 14 + 2 \\ & \boxed{0} \end{aligned}$$

$$(x^3 + 3x^2 - 15x + 18) \div (x + 6)$$

Is $(x+6)$ a factor?

$$\begin{array}{r|rrrr} -6 & 1 & 3 & -15 & 18 \\ & & -6 & 18 & -18 \\ \hline & 1 & -3 & 3 & \boxed{0} \end{array} \text{ Yes!}$$

Review: Find the factors and zeros of

$$x^3 + 2x^2 - 3x - 6 = 0$$

Factors $x^2(x+2) - 3(x+2) = 0$

$$(x^2 - 3)(x+2) = 0$$

Zeros $x^2 - 3 = 0$ $x+2 = 0$

$$x = \pm\sqrt{3} \quad x = -2$$

If $(x+2)$ is a factor of $f(x)=2x^3+x^2-5x+2$
 Find all linear factors and zeros

$$\begin{array}{r|rrrr} -2 & 2 & 1 & -5 & 2 \\ & & -4 & 6 & -2 \\ \hline & 2 & -3 & 1 & 0 \end{array} \quad \text{Yes!}$$

$$(x+2)(2x^2-3x+1)=0$$

$$(2x^2-2x-x+1)$$

$$2x(x-1)-1(x-1)$$

$$(x+2)(2x-1)(x-1)=0$$

$$\left\{ \begin{array}{l} 2 \\ -2 \quad -1 \end{array} \right| -3$$

Factors

$$x = -2, \frac{1}{2}, 1$$

zeros

Verify that $x = -3$ is a root of $f(x) = x^3 - 19x - 30$,
then find all linear factors and zeros

$$f(x) = x^3 - 8x^2 + 4x + 48$$

A) Is $(x-4)$ a factor?

B) What are all the linear factors

C) What are the roots?

$$(x-1) \quad x^3 + 5x^2 + 2x - 8$$

